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Review

Recent Progress on the Maxwell's Equations for Describing a Mechano-Driven Medium System with Multiple Moving Objects/Media

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Abstract — Maxwell's equations for a mechano-driven media system (MEs-f-MDMS) have been used to characterize the electromagnetism of multi-slow-moving media that may be accelerated with complex trajectories. Such an approach starts from the integral forms of the four physics laws and is different from the classical approach of using the Lorentz transformation for correlating the electromagnetic phenomena observed in two inertial reference frames with relative motion. The governing equations inside the moving object/medium are the MEs-f-MDMS, and those in vacuum are the classical Maxwell's equations; the full solutions of both reconcile at the medium surface/interface and satisfy the boundary conditions. This paper reviews the background, physical principle, and mathematical derivations for formulating the MEs-f-MDMS. Strategies are also presented for mathematically solving the MEs-f-MDMS. The unique advances made by the MEs-f-MDMS have been systematically summarized, as are their potential applications in engineering. We found that the Lorentz transformation is perfect for treating the electromagnetic phenomena of moving point charges in vacuum; however, for moving objects, the covariance of Maxwell's equations may not hold, and use of the MEs-f-MDMS may be required if the velocity is low. Finally, recent advances for treating the boundary conditions at the nanoscale without assuming an abrupt boundary are also reviewed.

Keywords — Maxwell's equations for a mechano-driven medium system (MEs-f-MDMS), Maxwell's equations, Lorentz transformation, Noninertia reference frame, Special relativity, Triboelectric nanogenerator (TENG).

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I. Introduction

Studying the electrodynamics of a moving medium has long-lasting interest and importance. For a general medium that moves with a uniform speed along a straight line, it is sufficient to use the standard "differential" Maxwell equations (MEs) and the approximate Minkowski constitutive equations for describing its electromagnetic behavior [1]–[3]. Using the Lorentz transformation, the electromagnetic fields observed in a moving frame (S') can be derived from a nonmoving observer's reference frame (S) by preserving the covariance of the MEs [3], [4]. This is the standard and well-received case of special relativity in classical electrodynamics, which is further shown in Figure 1(a). Special relativity concerns the same electromagnetic phenomenon as observed by two independent observers located in two inertial reference frames that have a relative movement at a constant speed, where the entire space is either vacuum or filled with medium without moving objects or boundaries [4], [5]. For example, an observer named Alice is in a moving inertial frame S' that moves at a velocity $\boldsymbol{v}_0$ relative to the nonmoving frame S (Lab frame). If there is a point charge $+q$ that is at rest in the S frame, a second observer (Bob) in the rest inertial frame S observes only a Coulomb field. For Alice, the point charge is moving at a relative velocity of $-\boldsymbol{v}_0$, so she will detect not only an electric field but also a magnetic field as caused by the moving charge $+q$ [5], [6]. The magnetic field and electric field observed by Alice ($\boldsymbol{B}'$, $\boldsymbol{E}'$) and Bob ($\boldsymbol{B}$, $\boldsymbol{E}$) are correlated by the Lorentz transformation under the assumption of the covariance of the governing equations.

To calculate the electromagnetic fields of a moving medium, the constitutive relations of materials that must be known and treated as supplemental conditions to solve the MEs in the relevant matter. Minkowski's views are grounded on an assumption that the properties of the medium and the corresponding constitution equations in the rest inertial frame remain the same. These views have two require-

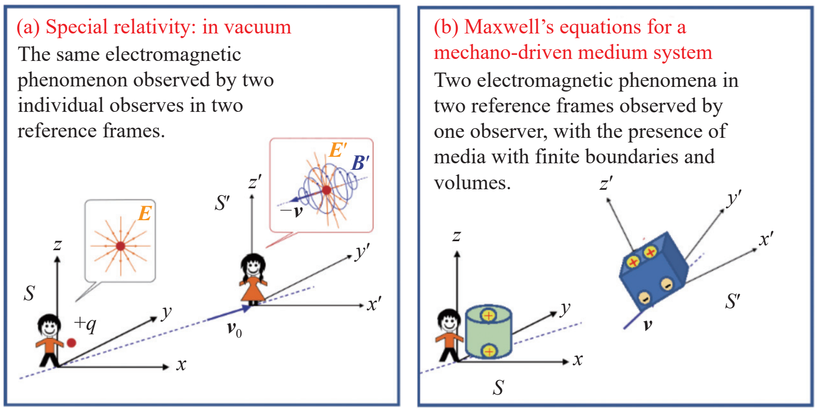

Figure 1 Two approaches for dealing with the electrodynamics of a moving medium. (a) Special relativity theory is about the experience of two independent observers, Bob and Alice, who are located in different reference frames (Lab frame, Moving frace) that are relatively moving at a constant velocity and along a straight line. Bob and Alice observe the same electromagnetic phenomenon occurring in vacuum space but with different measurement results. Such an approach is most effective for describing the electrodynamics in the universe. (b) Maxwell's equations for a mechano-driven media system (MEs-f-MDMS) is about one observer who is observing two electromagnetic phenomena, which are associated with two moving media located in the two reference frames that may relatively move at $v \ll c$. In general, the media/objects have sizes and boundaries, and they may move with acceleration along complex trajectories as driven by an external force. Such theory is most effective for engineering applications, but it can go beyond [6]. We need to point out that special relativity may not be easily adopted for describing the case shown in (b) due to the change in the speed of light across the medium boundary.

ments: movement with uniform speed along a straight line or in an inertial reference frame, and the corresponding constitution equations are predetermined [3], [4]. However, if the medium moves along a complex trajectory with acceleration and the velocity could be a function of time and position for shape-deformable materials or liquid, it is mathematically impossible to describe the electromagnetic fields of the moving medium in this case [7]. Such a case occurs frequently in engineering applications.

In general, there are two fundamental approaches for developing the electrodynamics of a moving medium (see Figure 2). The first method is through Einstein's relativity and Minkowski constitutive equations, forming the basis of field theory [3], [8]. The relativity approach works extremely well for describing the electromagnetic behavior in vacuum, especially for the universe. The second approach is based on the Galilean transformation, $x' = x - v_0 t$, $t' = t$, in which the space and time remain independent [9], [10]. Therefore, there exists an absolute space, and all inertial frames share a universal time scale, conclusions that are essentially distinct from those of special relativity, but it works well for engineering applications. Correspondingly, Galilean electromagnetism has been developed for describing the electromagnetic phenomenon of a charged medium moving at nonrelativistic speeds, which has been developed for over 60 years. According to some researchers, Galilean electromagnetism is not an alternative to special relativity but is precisely its low-velocity limit in classical electromagnetism [10]. Galilean electromagnetics mainly includes two quasistatic limits of MEs: the magneto-quasistatic (MQS) limit, which neglects the displacement current, and the electroquasi-static (EQS) limit, which ignores the magnetic induction [9], [10]. The former is a space-like limit with $E \ll cB$, while the latter is a time-like limit with $E \gg cB$. This second approach is based on the Newton's absolute space-time point of view. We recently developed Maxwell's equations for a mechano-driven media system (MEs-f-MDMS) (Figure 2) [11]–[13], which describes the electromagnetic behavior of media that move along a complex trajectory with arbitrary velocities by neglecting relativistic effects (Figure 3). Our goal is to further implementation of these approaches in engineering applications, as shown in Figure 1(b).

The initial motivation for developing MEs-f-MDMS was to quantify the output power and electromagnetic radiation produced by a triboelectric nanogenerator (TENG), which was invented in 2012 for converting mechanical energy into electric energy via the contact-electrification effect [14]–[16]. The relative movement of the dielectric media and electrodes makes the generated electric fields change under an external mechanical excitation. The driving force for the TENG is the displacement current [17]–[21]. Recently, the operation frequency of TENGs has reached as high as MHz, leading to the production of electromagnetic radiation [22], [23]. Some experiments have found that the converted electric power/signal can be transmitted wirelessly for a distance of 5 m under sea water, illustrating the possibility of wireless communication through the electromagnetic wave of TENGs, which makes us rethink the physical nature and influential factors behind these electromagnetic phenomena [24].

Although the MEs-f-MDMS was inspired by the experiments from TENGs, its impact and applications are expected to extend far beyond TENGs, especially considering the additional information that can be derived from the near field electrodynamics for virtual reality, control, sensing and feedback [11], [13], [25]. For the far field, MEs-f-MDMS can be used to construct electromagnetic images using the phase information from the reflected wave of a moving object.

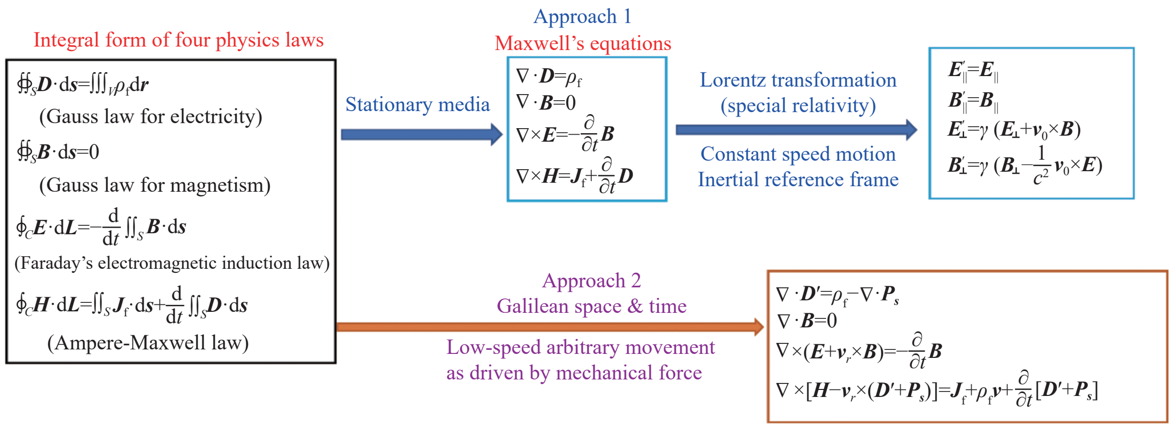

Figure 2 Two fundamentally different approaches for developing the electrodynamics of a moving media system: special relativity through the Lorentz transformation for electromagnetic phenomena of point charges in vacuum space and the MEs-f-MDMS directly derived from the integral forms of the four physics laws in Galilean space and time for the case of moving media with specific sizes and shapes and even acceleration. This is probably the most effective approach for engineering applications.

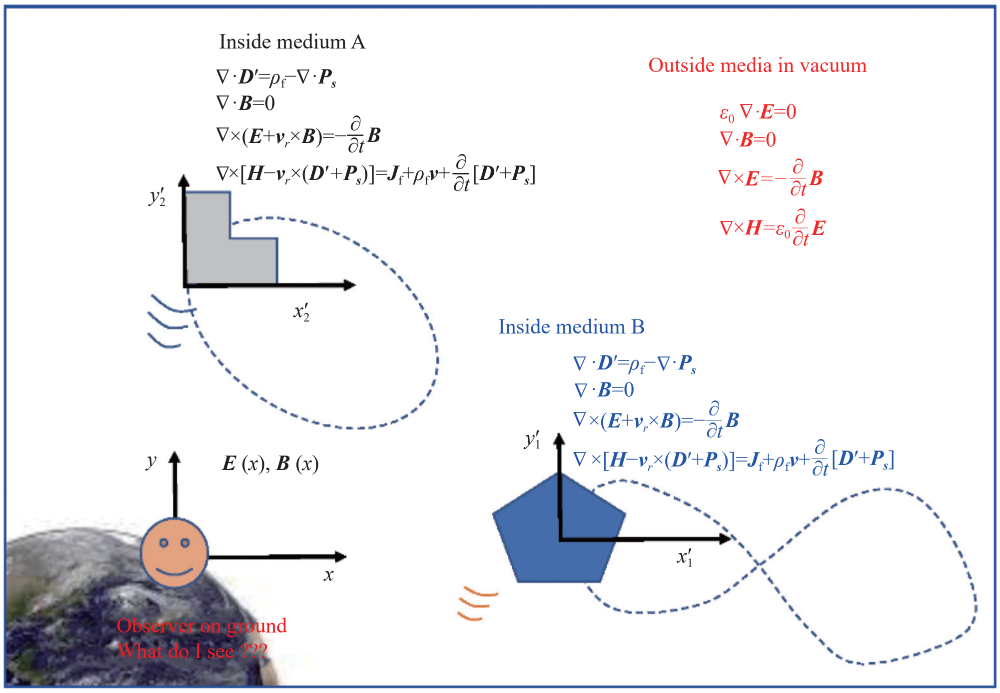

Figure 3 Schematic diagram showing the observation of several electromagnetic events that may move following complex trajectories in the Lab frame by an observed. The governing equations for each region are stated. The goal of MEs-f-MDMS is to describe the electromagnetic behavior in this system that occurs frequently in engineering [5], [11].

In this review, we present the background, motivations, physical approaches, and mathematical approaches for developing the theoretical framework of the MEs-f-MDMS. Coupled with mechanical force-electric-magnetic fields, the derived MEs-f-MDMS elucidate the dynamics of the electromagnetic field for a more general case, in which the moving medium displays a time-dependent volume, shape, and boundary as well as an arbitrary slow-moving velocity field $\boldsymbol{v}(r,t)$ in a noninertial frame. We first introduce an expansion to the displacement vector by considering the relative motion of the dielectric objects, which is termed the mechanoinduced polarization $\boldsymbol{P}_s$, and describe its physical meanings. Second, we derive the MEs-f-MDMS starting from the integral forms of the four physics laws and the corresponding boundary conditions. Third, we examine Faraday's law of electromagnetic induction to include Feynman's "anti-flux-rule" examples [26], and the most updated version of the MEs-f-MDMS is presented, which is systematic, fully logical and consistent with classical Maxwell's equations. Its physical meaning and associated application strategies are clearly explained. Fourth, based on recent work in another field, we generalize the nanoscale electromagnetic boundary conditions and interface response functions if classical Maxwell's equations are applied to calculate the electrodynamics of nanoscale objects, such as the plasmonics of nanoparticles. The nanoscale boundary conditions are derived by using the integral Maxwell's equations by constructing the dielectric transition layer across the interface between the two materials. Finally, key conclusions and potential impacts for the MEs-f-MDMS are summarized, and a summarizing perspective is presented.

II. Moving Medium is a Lot More than an Aggregation of Point Charges!

If a point charge moves at an arbitrary speed in vacuum without the presence of any boundaries, its electric field and magnetic fields can be calculated using the Liénard-Wiechert potentials [27], [28]. The electric field of the moving charge contains two parts: the generalized Coulomb field that is not dependent on the acceleration (also known as the velocity field) and the radiation field that is proportional to the acceleration. Note that only the acceleration fields represent true radiation. The free charge distribution and the instantaneous current produced by a group of moving point charges are represented by

$$\rho_{\mathrm{f}} = \sum_i q_i \delta(\boldsymbol{r} - \boldsymbol{r}_i(t)) \tag{1a}$$

$$\boldsymbol{J}_{\mathrm{f}} = \sum_i q_i \boldsymbol{v}_i(t) \delta(\boldsymbol{r} - \boldsymbol{r}_i(t)) \tag{1b}$$

where $\boldsymbol{r}_i(t)$ and $\boldsymbol{v}_i(t)$ are the instantaneous position and moving velocity of the point charge q_i. The distribution of the field in space can be calculated by substituting equations (1a) and (1b) into the Liénard-Wiechert potentials.

A point charge is just a point without volume and boundary. Lorentz transformation is ideal for treating the electrodynamics of moving point charges in vacuum. However, a medium is not just an aggregation of point charges but is composed of atoms with special symmetry, geometry, shape and size. Due to its unique crystal structure and chemistry, a medium typically has dielectric, electrical, magnetic and elastic properties. Therefore, it has different electrical, optical, thermal and mechanical properties. For a moving medium that has electrostatic charges on surfaces, the approach of Liénard-Wiechert potentials cannot be utilized to calculate its electromagnetic fields. This is a reason why we expand the MEs to study the electromagnetic behavior of the motion media/object, to yield a system that could be time- and even space-independent.

To represent the characteristics of media/materials in electromagnetic theory, electromagnetic excitation is described by electric ($\boldsymbol{P}$) and magnetic ($\boldsymbol{M}$) polarizations, respectively and was first developed over a century ago. Deepening our understanding of the electrodynamics of moving media is an important research topic that is generally advanced through the macroscopic MEs and Minkowski material equations [1]–[3]. In general, inhomogeneities of the velocity of a moving medium, if it is shape deformable or in a liquid state, generate an inhomogeneity of the refractive index. If a medium is in a static state, the propagation of electromagnetic waves passing through it is governed by three parameters: permittivity (ε), permeability (μ), and conductivity (σ). However, each of these parameters depends heavily on the frequency of the electromagnetic wave we are considering. Electromagnetic waves with different frequencies travel at the same speed in vacuum, but they interact with media differently due to dielectric dispersion. Therefore, the variations in the permittivity, permeability and/or refractive index lead to the scattering of electromagnetic radiation of the medium.

III. Some Considerations About the Special Relativity for the Medium Case

1. Mechanoinduced polarization

Special relativity was proposed based on two hypotheses: i) The laws of physics take the same form in every inertial frame; and ii) The speed of light in vacuum is the same in every inertial frame. Special relativity is the theory of how different observers, moving at constant velocity with respect to one another, report their experience of the same physical event. General relativity addresses the same issue for observers whose relative motion is completely arbitrary. Therefore, the *Lorentz transformation is an exact calculation if all of the electromagnetic phenomena are in vacuum.*

A key quantity in the Lorentz transformation is the speed of light c because this parameter unifies space and time as follows:

$$x' = \gamma_0(x - v_0 t),\ \ y' = y,\ \ z' = z \tag{1c}$$

$$t' = \gamma_0\left(t - x v_0 / c_0{}^2\right) \tag{1d}$$

$$\gamma_0 = 1/\left(1 - v_0{}^2/c_0{}^2\right)^{1/2} \tag{1e}$$

or

$$x = \gamma_0(x' + v_0 t'),\ \ y = y',\ \ z = z' \tag{2a}$$

$$t = \gamma_0\left(t' + x' v_0 / c_0{}^2\right) \tag{2b}$$

If all of the moving point charges are in vacuum, c_0 should be the speed of light in vacuum, and the situation should be easily described because a point charge has no volume and boundary, and the system can be represented by a set of points with charge density and related current [27], [29]. The resulting MEs are covariant because of the use of the Lorentz transformation.

However, the situation is complex if there is medium. If the entire space is filled with a uniform medium so that the speed of light would be $c_{\mathrm{m}} = c_0/n$, where n is the refraction index, the corresponding Lorentz transformation inside the medium would be [30]

$$x'_{\mathrm{m}} = \gamma_{\mathrm{m}}(x - v_0 t),\ \ y'_{\mathrm{m}} = y,\ \ z'_{\mathrm{m}} = z \tag{3a}$$

$$t'_{\mathrm{m}} = \gamma_{\mathrm{m}}\left(t - x v_0 / c_{\mathrm{m}}{}^2\right) \tag{3b}$$

$$\gamma_{\mathrm{m}} = 1/\left(1 - {v_0}^2/{c_{\mathrm{m}}}^2\right)^{1/2} \tag{3c}$$

or

$$x = \gamma_{\mathrm{m}}\left(x'_{\mathrm{m}} + v_0 t'_{\mathrm{m}}\right),\ \ y = y'_{\mathrm{m}},\ \ z = z'_{\mathrm{m}} \tag{4a}$$

$$t = \gamma_{\mathrm{m}}\left(t'_{\mathrm{m}} + x'_{\mathrm{m}} v_0/{c_{\mathrm{m}}}^2\right) \tag{4b}$$

Equations (3) and (4) hold if the medium is isotropic, and the dielectric constant and magnetic permittivity are constants, so that the speed of light in the medium is independent of the observation frame.

Now we consider another case, in which the space in the $x' > 0$ zone in the moving frame S' is filled with a uniform and linear dielectric medium, and it is moving at a constant velocity v_0. The zone at $x' < 0$ is vacuum. How the Lorentz transformation would be constructed to ensure the space and time continuous at the medium boundary? In practical engineering applications, where part of the space is filled with dielectric media/objects and part is a vacuum, what would be the correct expression of Lorentz transformation? How do we express the unification of space and time in such a case? These questions are investigated here.

If we consider the dilation of time and contraction of length in relativity, an expanded Lorentz transformation for the space and time $(\boldsymbol{r}'_{\mathrm{m}}, t'_{\mathrm{m}})$ inside the moving medium in the $x' > 0$ zone could be formulated as

$$x = \gamma_0(x'_{\mathrm{m}} + v_0 t'_{\mathrm{m}}),\ \ y = y'_{\mathrm{m}},\ \ z = z'_{\mathrm{m}} \tag{5a}$$

$$t = \gamma_0(t'_{\mathrm{m}} + x'_{\mathrm{m}} v_0/{c_{\mathrm{m}}}^2) \tag{5b}$$

or

$$x'_{\mathrm{m}} = {\gamma_{\mathrm{m}}}^2(x - v_0 t)/\gamma_0,\ \ y'_{\mathrm{m}} = y,\ \ z'_{\mathrm{m}} = z \tag{6a}$$

$$t'_{\mathrm{m}} = {\gamma_{\mathrm{m}}}^2(t - x v_0/{c_{\mathrm{m}}}^2)/\gamma_0 \tag{6b}$$

Here, we also assume that S' is moving along $+x$-axis at a speed of v_0. Equation (5) not only satisfies the continuation of the space and time at the $x'_{\mathrm{m}} = 0$ boundary but also approaches the associated standard Lorentz transformations by replacing $c_{\mathrm{m}} \to c_0$ and $c_0 \to c_{\mathrm{m}}$ for the cases of the entire space being vacuum and filled with a medium, respectively. However, the symmetry preserved between equation (1) and equation (2) is not maintained in (5) and (6). Specifically, it does not hold by simply replacing $v_0 \to v_0$! Because of the presence of the medium boundary. Therefore, in the moving object case, the covariance of the MEs may not be preserved for correlating the electromagnetic phenomenon observed in the S' frame (the frame in which the medium is stationary) with that observed in the S frame (the observer's frame) because of the change in space symmetry by the object boundaries and dielectrics (see Figure 1(b)) [31]. The validity of equations (5) and (6) remains to be further studied, and it is just a proposal here.

2. Are Maxwell's equations covariant for a moving media/object system?

The above discussions may indicate that the covariance of the MEs do not hold if there is a complex media distribution in space (see Figure 1(b)) [31]. As indicated in [31], it would be correct to state that Maxwell's equations perfectly fit to be Lorentz-covariant if the point charge related electromagnetic phenomena and observations are made in vacuum. Otherwise, the covariance may not hold.

Furthermore, we now consider the constitutive relation in a realistic medium. If we ignore the dependence of dielectric permittivity on the momentum transfer term $\boldsymbol{q}$, for a simple linear medium, in the frequency space, we obtain

$$\boldsymbol{D}(\boldsymbol{r},\omega) = \varepsilon(\omega)\,\boldsymbol{E}(\boldsymbol{r},\omega) \tag{7a}$$

In time space, and using the inverse Fourier transformation, equation (7a) becomes [32]

$$\boldsymbol{D}(\boldsymbol{r},t) = \int_{-\infty}^{\infty} \varepsilon(t-t')\,\boldsymbol{E}(\boldsymbol{r},t')\,\mathrm{d}t' \tag{7b}$$

This means that if we consider the anisotropic property of a dielectric medium and its frequency dependence, the constitutive relationship between the displacement vector $\boldsymbol{D}$ and the electric field $\boldsymbol{E}$ cannot be simply treated as $\boldsymbol{D}(\boldsymbol{r},t) = \varepsilon\boldsymbol{E}(\boldsymbol{r},t)$ unless ε is a constant. Therefore, as a general case, the covariance of the MEs holds exactly in vacuum but may not hold exactly in a dielectric medium unless the medium's property is independent of the excitation frequency ω, which means that there is no dispersion dependence [3], [4], [33]. Such cases may not be true for applying to practical materials. For an inhomogeneous material, such as ferroelectric or piezoelectric crystals, the dielectric $\varepsilon(\omega)$ is described using a tensor, depending on the orientation of the medium. Therefore, *the covariance of the MEs holds exactly for the electromagnetic phenomena occurring in vacuum.*

IV. Medium Polarization

1. Polarization induced by an electric field

It is well established that a dielectric material can be polarized if it is placed in an external electric field. The generalized medium polarization $\boldsymbol{P}$ is an average description of the macroscopic structure for a linear medium, which is expressed as

$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E} \tag{8}$$

where ε_0 and χ represent the permittivity of vacuum and the electric susceptibility of the medium, respectively. Here, $\boldsymbol{E}$ is the total local electric field. The potential of a polarized dielectric is the same as that created by a volume charge density $\rho_b = -\nabla\cdot\boldsymbol{P}$ plus a surface charge density $\boldsymbol{\sigma}_b = -\boldsymbol{P}\cdot\widehat{\boldsymbol{n}}$ [27], [29].

2. Mechanoinduced polarization

In classical electromagnetism, the medium boundary and shape are time-independent, but the whole medium/object can move with a uniform speed along a straight line. In engineering applications, media can move with acceleration along complex trajectories, and their shapes may vary with time. The surfaces of the media may have electrostatic charges, so their relative movement may introduce an additional polarization term. Therefore, we need to find an effective approach to describing such a case.

Taking the triboelectric nanogenerator (TENG) as an example, the device needs at least one moving medium to generate electrostatic charges caused by contact-electrification and excited by an external mechanical force. As a result, the media will be polarized due to the electric field generated by the electrostatic charges. This polarization is essentially different from the $\boldsymbol{P}$ that arises due to an external electric field. In fact, variations in the moving medium object and medium shape lead to not only a local time-dependent charge density but also a local "virtual" electric current density. To account for these phenomena, an additional term $\boldsymbol{P}_s$ termed mechano-driven polarization is introduced [11]:

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} + \boldsymbol{P}_s = \varepsilon_0(1+\chi)\boldsymbol{E} + \boldsymbol{P}_s \tag{9}$$

where the first term $\varepsilon_0\boldsymbol{E}$ is due to the field created by the free charges, which is the field for exciting the media. The vector $\boldsymbol{P}$ is the medium polarization, and it is responsible for the screening effect of the medium to the external electric field. The added term $\boldsymbol{P}_s$ is mainly due to the existence of surface electrostatic charges and the time variation in boundary shapes. The charges that directly contribute to the term $\boldsymbol{P}_s$ are neither free charges nor polarization-induced charges; instead, they are intrinsic surface-bound electrostatic charges as introduced by external mechanical triggering to the media. We note that since the research object is not considered a point charge, the existence of a new term, $\boldsymbol{P}_s$, which is also called the Wang term, needs to be considered. This term applies to both isotropic media and anisotropic media and is necessary for developing the theory of TENGs [6], [21]. The corresponding space charge density is

$$\rho_s = -\nabla\cdot\boldsymbol{P}_s \tag{10a}$$

the surface electrostatic charge density is $\sigma_s = \boldsymbol{n}\cdot\boldsymbol{P}_s$, and the displacement current density contributed by the bond electrostatic charges owing to medium movement is

$$\boldsymbol{J}_s = \frac{\partial}{\partial t}\boldsymbol{P}_s \tag{10b}$$

For easy notation, we define $\boldsymbol{D}'$ to represent the field-induced displacement vector term

$$\boldsymbol{D}' = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} \tag{11a}$$

Please note the prime does not stand for a moving reference frame as in classical electrodynamics! The total displacement vector is

$$\boldsymbol{D} = \boldsymbol{D}' + \boldsymbol{P}_s \tag{11b}$$

The calculation of $\boldsymbol{P}_s$ is as reported previously [6], [21].

V. Deriving Maxwell's Equations for a Mechano-driven System

Two methods have been developed to address the electrodynamics of moving media: relativistic electrodynamics and Galilean electromagnetism [1]–[3], [9], [10]. Using the Lorentz transformation, the electromagnetic behavior of a moving medium is described using classical MEs. However, the preconditions are that the velocity of the moving media is uniform in the inertia frame, and the constitutive relationships of the moving media are known. Minkowski's approach is taken as the formal method for a moving medium. The second method uses Galilean Electromagnetism. Note that Galilean electromagnetism is not an alternative to special relativity but is precisely its low-velocity limit in classical electromagnetism [10]. Under quasistatic approximation, this method is utilized for magnetic-dominated and electric-dominated systems. Similar to Minkowski electrodynamics, this approach applies to the case in which the object moves with uniform speed along a straight line.

However, in practice, the medium always moves with acceleration, or more generally, several media move at complex velocities along various trajectories in a noninertial frame, and some efforts focus on the scattering, reflection and transmission of electromagnetic waves from slow-moving media [34]–[36]. To develop an effective approach for solving for the electromagnetism of moving objects, we start from the integral forms of the four physics laws: a) Gauss's law for electricity, b) Gauss's law for magnetism, c) Faraday's electromagnetic induction law (Lenz law), and d) Ampere-Maxwell law. Since the moving velocity of the object is $v \ll c_0$ and the physical dimension we consider is much smaller than the distance traveled by light within the duration of the event, the Galilean transformation is an excellent approximation [37]. In such a case, we can neglect the relativistic effect so that the final formulation can be more amenable for applying to engineering problems [6], [11].

$$\oiint_S \boldsymbol{D}'\cdot \mathrm{d}\boldsymbol{s} = \iiint_V \rho_\mathrm{f}\mathrm{d}\boldsymbol{r} \tag{12a}$$

$$\oiint_S \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{s} = 0 \tag{12b}$$

$$\oint_C \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{L} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{s} \tag{12c}$$

$$\oint_C \boldsymbol{H}\cdot \mathrm{d}\boldsymbol{L} = \iint_S \boldsymbol{J}_\mathrm{f}\cdot \mathrm{d}\boldsymbol{s} + \frac{\mathrm{d}}{\mathrm{d}t}\iint_S \boldsymbol{D}'\cdot \mathrm{d}\boldsymbol{s} \tag{12d}$$

where ρ_f is the density of free charges in space, and $\boldsymbol{J}_\mathrm{f}$ is the current density. The surface integrals for $\boldsymbol{B}$ and $\boldsymbol{D}'$ are for a surface S that is defined by a closed loop C, and they are the magnetic flux and displacement field flux, respectively. The integral forms are provided based on the physics phenomena that have been observed experimentally: the total electric flux through a closed surface is the total charges contained inside (equation (12a)); the total magnetic flux through a closed surface is zero (equation (12b)); the decreasing rate of the magnetic flux through an open surface is the circulation of the electric field around its looped edge (equation (12c)); the total electric current through an open surface plus the changing rate of the electric flux through the surface is the circulation of the magnetic field around its looped edge (equation (12d)).

There are two more laws to consider. The charge conservation law expressed as [11]

$$\oiint_S \boldsymbol{J}_{\mathrm{f}} \cdot \mathrm{d}\boldsymbol{s} + \frac{d}{dt}\iiint_V \rho_{\mathrm{f}} \mathrm{d}\boldsymbol{r} = 0 \tag{13}$$

indicates that the total current flowing into a closed surface equals the changing rate of the total charges inside. The final law is the Coulomb-Lorentz force, which describes the force experienced by a particle with charge q and moving velocity $\boldsymbol{v}$ in the presence of an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$, and it can be expressed as

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{14}$$

Maxwell's equations (12), the continuity equation of charge (13), and the Coulomb-Lorentz force equation (14) establish the foundation of classical electrodynamics.

The conditions for the classical Maxwell's equations to hold exactly are that the boundaries and distribution configurations of the dielectrics in space are fixed or time-independent [28], [29]. However, such a condition is rarely mentioned in textbooks, which might lead to a misunderstanding that the MEs can be utilized to describe any and all of the electromagnetic phenomena. For the electrodynamics of a moving medium, the primary method is to solve Minkowski's equations that were derived based on the principle of relativity. However, this method requires that the constitutive relationships of the moving media are redefined, and these equations hold only for the case that the media is moving at a constant velocity along a straight line [1]–[3]. In a more general case, a medium/object moves along a complex trajectory or there are several different media that move at complex velocities along various trajectories. Solving the above problems is extremely difficult using the Lorentz transformation, which describes the electromagnetic fields from the comoving frame to the observation frame via a specific coordinate transformation by correlating space and time. Our goal is to express all of the observed fields in frame S with a systematic consideration of the movement of the objects and their interactions without going through the coordination transformation so that the fields in frame S can be directly calculated: $\boldsymbol{E}(\boldsymbol{r},t)$, $\boldsymbol{B}(\boldsymbol{r},t)$, $\boldsymbol{H}(\boldsymbol{r},t)$, $\boldsymbol{D}'(\boldsymbol{r},t)$, $\rho(\boldsymbol{r},t)$, $\boldsymbol{J}_{\mathrm{f}}(\boldsymbol{r},t)$. Using the flux theorem in field theory [6], [11]:

$$\oiint_S \boldsymbol{D}' \cdot \mathrm{d}\boldsymbol{s} = \iiint_V \rho_{\mathrm{f}} \mathrm{d}\boldsymbol{r} \tag{15a}$$

$$\oiint_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = 0 \tag{15b}$$

$$\oint_C \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{L} = -\iint_S \frac{\partial}{\partial t}\boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} + \oint_C (\boldsymbol{v} \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{L} \tag{15c}$$

$$\oint_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{L} = \iint_S (\boldsymbol{J}_{\mathrm{f}} + \rho_{\mathrm{f}}\boldsymbol{v}) \cdot \mathrm{d}\boldsymbol{s} + \iint_S \frac{\partial}{\partial \mathrm{t}}\boldsymbol{D}' \cdot \mathrm{d}\boldsymbol{s} - \oint_C (\boldsymbol{v} \times \boldsymbol{D}') \cdot \mathrm{d}\boldsymbol{L} \tag{15d}$$

where $\boldsymbol{v}(\boldsymbol{r},t)$ is the velocity field of the moving medium that is time and space dependent. Using Stokes's theorem and divergence theorem, the governing equations for the space inside the moving medium are provided in differential form as follows:

$$\nabla \cdot \boldsymbol{D}' = \rho_{\mathrm{f}} \tag{16a}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{16b}$$

$$\nabla \times [\boldsymbol{E} - \boldsymbol{v} \times \boldsymbol{B}] = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{16c}$$

$$\nabla \times [\boldsymbol{H} + \boldsymbol{v} \times \boldsymbol{D}'] = \boldsymbol{J}_{\mathrm{f}} + \rho_{\mathrm{f}}\boldsymbol{v} + \frac{\partial}{\partial \mathrm{t}}\boldsymbol{D}' \tag{16d}$$

The results are consistent with those reported previously [34], [35]. It is important to note that the terms $\boldsymbol{v} \times \boldsymbol{B}$ and $\boldsymbol{v} \times \boldsymbol{D}'$ are the sources of generated electromagnetic waves due to media movement even if their change rate is zero. This observation is new, and its application will be explored in the near future. Note that if the velocity decreases to zero, equations (16a)–(16d) become the classical Maxwell's equations [38]. For the space outside the moving medium, the terms containing $\boldsymbol{v}$ drop out, and equations (16a)–(16d) resume the standard format of the MEs. Both sets of mathematical solutions inside and outside of the medium satisfied the boundary conditions, which can be derived from equations (15a) and (15d) [6], [11]

$$[\boldsymbol{D}_2' - \boldsymbol{D}_1'] \cdot \boldsymbol{n} = \sigma_{\mathrm{f}} \tag{17a}$$

$$[\boldsymbol{B}_2 - \boldsymbol{B}_1] \cdot \boldsymbol{n} = 0 \tag{17b}$$

$$\boldsymbol{n} \times [\boldsymbol{E}_2 - \boldsymbol{E}_1 - \boldsymbol{v} \times (\boldsymbol{B}_2 - \boldsymbol{B}_1)] = 0 \tag{17c}$$

$$\boldsymbol{n} \times [\boldsymbol{H}_2 - \boldsymbol{H}_1 + \boldsymbol{v} \times (\boldsymbol{D}_2' - \boldsymbol{D}_1')] = \boldsymbol{K}_s + \sigma_{\mathrm{f}}\boldsymbol{v}_s \tag{17d}$$

where ρ_{f} represents the surface free charge density, $\boldsymbol{K}_s$ is the surface current density, $\boldsymbol{v}_s$ is the moving velocity of the media parallel to the boundary surface, and $\boldsymbol{n}$ represents the surface normal direction.

The charge conservation law is given by

$$\nabla \cdot (\boldsymbol{J}_{\mathrm{f}} + \rho_{\mathrm{f}}\boldsymbol{v}) + \frac{\partial}{\partial t}\rho_{\mathrm{f}} = 0 \tag{18}$$

where $\rho_{\mathrm{f}}\boldsymbol{v}$ represents the local current generated by the free charges owing to medium movement.

In addition, the deformation of the medium geometry and medium movement produce a time-dependent charge density and an effective electric current density, which is represented by the mechano-driven polarization $\boldsymbol{P}_s$. We replace $\boldsymbol{D}'$ with $\boldsymbol{D}$=$\boldsymbol{D}'$+$\boldsymbol{P}_s$ in equations (16a)–(16d) [6], [11] and have

$$\nabla \cdot \boldsymbol{D}' = \rho_{\mathrm{f}} - \nabla \cdot \boldsymbol{P}_s \tag{19a}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{19b}$$

$$\nabla\times(\boldsymbol{E}-\boldsymbol{v}\times\boldsymbol{B})=-\frac{\partial}{\partial t}\boldsymbol{B} \tag{19c}$$

$$\nabla\times[\boldsymbol{H}+\boldsymbol{v}\times(\boldsymbol{D}'+\boldsymbol{P}_s)]=\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}+\frac{\partial}{\partial \mathrm{t}}[\boldsymbol{D}'+\boldsymbol{P}_s] \tag{19d}$$

Using these equations, the coupling between mechanical, electrical and magnetic performances and behaviors of the system can be systematically described. In (19d), the term $\boldsymbol{v}\times(\boldsymbol{D}'+\boldsymbol{P}_s)$ is the local induced magnetic field because of medium movement in the local electric field.

VI. Maxwell's Equations for a Mechano-Driven System Including Feynman's "Anti-flux" Examples

The mathematical expression of Faraday's law of electromagnetic induction is the flux rule: the reducing rate of the magnetic flux is the electromotive force (*emf* for short, distinguished from EMF, namely electromagnetic field). It can be expressed as

$$\xi_{emf}=-\frac{\mathrm{d}\Phi_{\boldsymbol{B}}}{\mathrm{dt}}=-\frac{\mathrm{d}}{\mathrm{dt}}\iint_{s(t)}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{s} \tag{20}$$

Equation (20) is the flux rule, which can be used to describe most of the electromagnetic phenomena, especially for power generation and electric motors. However, there are a few cases that appear to describe an "anti-flux-rule," as presented by Feynman [26]. Figure 4 shows such a case in which the circuit contains a rotating metal disc with a sliding needle at its edge. Once the disc rotates, the total magnetic flux that passes through the circuit does not change, so there should be no *emf* according to (20), but an *emf* does exist experimentally. This paradox was not clearly explained by Feynman. In his textbook, he said, "It must be applied to circuits in which the material of the circuit remains the same. When the material of the circuit is changing, we must return to the basic laws. The correct physics is always given by the two basic laws $\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}$, $\boldsymbol{F}=q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$" [26]. Therefore, a more detailed explanation is missing.

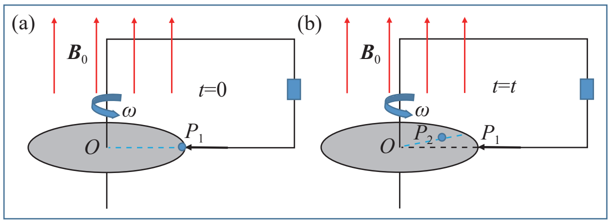

Figure 4 An example of the "anti-flux rule" as first presented by Feynman. A rectangular thin wire circuit that is stationary in a uniform magnetic field; one end slides on the edge of a rotating conductive disc. (a) The point charge starting moving from P_1 at t = 0, and (b) the point charge reaches point P_2 at t = t.

The "anti-flux-rule" is likely attributed to the path of the unit charge moving in the disc (as indicated by a blue dashed line) deviating from the original rectangular "circuit" (as indicated by the black dashed line in Figure 4(b)) along which the integral for calculating the magnetic flux is performed. As the charge enters the disc at point P_1 at its edge at t = 0, it moves along the radial direction to point P_2 as the disc rotates to t = t; its moving path is indicated by a blue dashed line. Therefore, the area defined by the two dashed lines in Figure 4(b) is the effective area of change in magnetic flux. This change in flux is due to the deviation of the unit charge transport path from that of the geometrical path as the disc rotates. This is caused by the existence of the large metal disc in the circuit, the rotation of which produces the observed *emf*. To include this argument officially in the equation, we first focus on the expansion of Faraday's law.

If there is no change in the circuit, using the flux theorem in field theory, equation (20) can be mathematically derived as [5], [12]

$$\xi_{emf}=-\frac{\mathrm{d}}{\mathrm{d}t}\iint_{s(t)}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{s}=-\iint_{s(t)}\left\{\frac{\partial}{\partial t}\boldsymbol{B}-\nabla\times[\boldsymbol{v}\times\boldsymbol{B}]\right\}\cdot\mathrm{d}\boldsymbol{s} \tag{21}$$

where $\boldsymbol{v}$ is the velocity at which the boundary surface moves. Equations (20) and (21) are supposed to be mathematically identical due to the definition of the looped circuit. However, equation (20) applies to the case in which there is no change in the closed circuit; for instance, there is no relative sliding between the wire and the disc. In (21), $\boldsymbol{v}$ means the moving velocity of the circuit, and it allows a flexible or changeable contact between the wire and the disc. Most importantly, if the circuit is not a closed loop, equation (21) should be used, but (20) cannot be utilized.

There are two integral forms of Faraday's law: one is given in (15c), and the other form is written as

$$\oint_C\boldsymbol{E}'\cdot\mathrm{d}\boldsymbol{L}=-\frac{\mathrm{d}}{\mathrm{d}t}\iint_C\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{s} \tag{22}$$

where $\boldsymbol{E}'$ represents the electric field in the rest frame of each segment d$\boldsymbol{L}$ of the path of integration. We now need to express $\boldsymbol{E}'$ in terms of the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ in the Lab frame. Using equation (21) into (22), it gives

$$\oint_C(\boldsymbol{E}'-\boldsymbol{v}\times\boldsymbol{B})\cdot\mathrm{d}\boldsymbol{L}=-\iint_{s(t)}\frac{\partial}{\partial t}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{s} \tag{23}$$

When the medium moves with an acceleration, the electromagnetic behavior of the moving medium becomes very complex. We take a unit charge q as an example. If the medium that carries the unit charge experiences an acceleration motion, the force acting on the unit charge includes the inertia force $\frac{\partial}{\partial t}(m\boldsymbol{v})$ in addition to the electromagnetic force, where m is the mass of the point charge. In this case, we have [5]

$$q\boldsymbol{E}'-\frac{\partial}{\partial t}(m\boldsymbol{v})=q\boldsymbol{E}+q\boldsymbol{v}_t\times\boldsymbol{B} \tag{24a}$$

where $\boldsymbol{v}_t$ is the total moving velocity of the unit charge in the S frame. $\boldsymbol{v}_t$ can be split into two components for a general case (see Figure 5): moving velocity $\boldsymbol{v}$ of the origin of the reference frame S', which is only time-dependent and can be viewed as a rigid translation, where $\boldsymbol{v}_r$ is the relative moving velocity of the point charge with respect to the reference frame S', which is space and time dependent.

$$\boldsymbol{v}_t = \boldsymbol{v}(t) + \boldsymbol{v}_r(\boldsymbol{r}, t) \tag{24b}$$

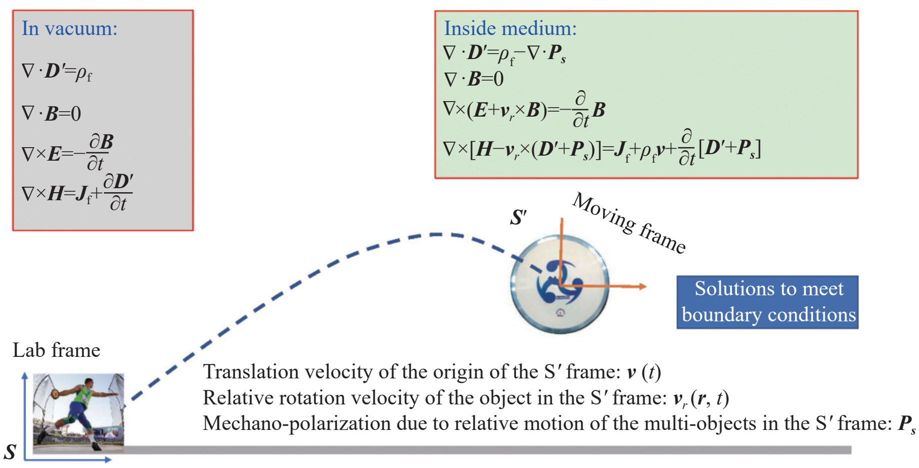

Figure 5 We use a flying disc to illustrate the applications of the MEs-f-MDMS for engineering purposes. The electromagnetic behavior inside the medium (the moving disc) is the MEs-f-MDMS, while that in vacuum is the classical ME; the solutions of the two sets of equations meet the boundary conditions at the medium interfaces/surface. $\boldsymbol{v}(t)$is the moving velocity of the origin of the S' reference frame; $\boldsymbol{v}_r(\boldsymbol{r}, t)$ is the relative movement velocity of the object in the moving reference frame; $\boldsymbol{P}_s$ is the polarization introduced due to the relative movement of the objects in the moving reference frame if there are more objects to be considered.

The space dependence of $\boldsymbol{v}_r$ represents the shape deformation and/or rotation of the medium, and the time dependence represents the local acceleration. Substituting equations (24a) and (24b) into (23), we obtain

$$\oint_C \left[\boldsymbol{E} + \boldsymbol{v}_r \times \boldsymbol{B} + \frac{1}{q}\frac{\partial}{\partial t}(m\boldsymbol{v})\right] \cdot \mathrm{d}\boldsymbol{L} = -\iint_s \frac{\partial}{\partial t}\boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} \tag{25}$$

There are two cases considered here [5]:

1) If a circuit is a thin wire so that the moving velocity $\boldsymbol{v}_r$ is parallel to the integral path $\mathrm{d}\boldsymbol{L}$, the term $[\boldsymbol{v}_r \times \boldsymbol{B}] \cdot \mathrm{d}\boldsymbol{L}$ vanishes, equation (25) leads to the standard MEs.

2) If the medium is a large piece so that the moving velocity $\boldsymbol{v}_r$ inside the medium is not parallel to the integral path $\mathrm{d}\boldsymbol{L}$, the $\boldsymbol{v}_r \times \boldsymbol{B}$ term remains. This describes the case on which we are focused in the following discussions. Using the Stokes theorem, equation (25) becomes

$$\nabla \times \left[\boldsymbol{E} + \boldsymbol{v}_r \times \boldsymbol{B} + \frac{1}{q}\frac{\partial}{\partial t}(m\boldsymbol{v})\right] = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{26a}$$

Since the movement of the origin of the reference frame S' that is affixed to the moving medium can be treated as a rigid translation (thus it is only time dependent), the expression for $\boldsymbol{v}(t)$ is given as $\nabla \times \left(\frac{\partial}{\partial t}(m\boldsymbol{v}(t))\right) = \frac{\partial}{\partial t}[m\nabla \times \boldsymbol{v}(t)] = 0$, so that the inertia force term drops out naturally in the differential equation, allowing us to obtain [5]

$$\nabla \times (\boldsymbol{E} + \boldsymbol{v}_r \times \boldsymbol{B}) = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{26b}$$

In equation (25), when the integral path C is intercepted by a bulk size medium, inside which the practical moving velocity of the point charge may not be parallel to the integral path within the conductive medium, the term $\boldsymbol{v}_r \times \boldsymbol{B}$ appears in the equation. It should be noted that the relative velocity $\boldsymbol{v}_r$ of the charge inside a medium may not be small in comparison to the moving velocity $\boldsymbol{v}(t)$ of the reference frame.

Now, we consider the case presented in Figure 4. If the magnetic field is time-independent, $\frac{\partial}{\partial t}\boldsymbol{B} = 0$, from (26b), we have $\nabla \times \boldsymbol{E} = -\nabla \times (\boldsymbol{v}_r \times \boldsymbol{B})$, which means that the movement of the medium in a magnetic field generates an electric field, the calculation is in agreement with experimental observations. However, the situation is different if we use the classical MEs; for which $\nabla \times \boldsymbol{E} = 0$, indicating that $\boldsymbol{E} = 0$ in apparent disagreement with experimental observations. Such discrepancy is because medium motion was not considered in classical MEs! This is another reason that we need to expand the MEs.

Equation (26b) is the expanded format of Faraday's law of electromagnetic induction, and it includes the cases of Feynman's "anti-flux-rule" examples. Similarly, the expansion of Ampere-Maxwell's law for a macroscopic media system that moves with acceleration could be straightforward. If we consider the symmetry between electricity and magnetism as well as the equivalence of the two fields and uses (19d) as an example, Ampere-Maxwell's law can be expanded as

$$\nabla \times (\boldsymbol{H} - \boldsymbol{v}_r \times \boldsymbol{D}) = J_\mathrm{f} + \rho_\mathrm{f}\boldsymbol{v} + \frac{\partial}{\partial t}D \tag{27}$$

Therefore, the electrodynamics inside the media can be described by [5], [13]

$$\nabla\cdot\boldsymbol{D}'=\rho_{\mathrm{f}} \tag{28a}$$

$$\nabla\cdot\boldsymbol{B}=0 \tag{28b}$$

$$\nabla\times(\boldsymbol{E}+\boldsymbol{v}_r\times\boldsymbol{B})=-\frac{\partial}{\partial t}\boldsymbol{B} \tag{28c}$$

$$\nabla\times(\boldsymbol{H}-\boldsymbol{v}_r\times\mathbf{D}')=\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}+\frac{\partial}{\partial t}\boldsymbol{D}' \tag{28d}$$

The above equations are the MEs-f-MDMS for a media system that moves with an arbitrary but low velocity even with acceleration. If motion-induced mechanopolarization is considered, the above equations are transformed into [5], [13]

$$\nabla\cdot\boldsymbol{D}'=\rho_{\mathrm{f}}-\nabla\cdot\boldsymbol{P}_s \tag{29a}$$

$$\nabla\cdot\boldsymbol{B}=0 \tag{29b}$$

$$\nabla\times(\boldsymbol{E}+\boldsymbol{v}_r\times\boldsymbol{B})=-\frac{\partial}{\partial t}\boldsymbol{B} \tag{29c}$$

$$\nabla\times[\boldsymbol{H}-\boldsymbol{v}_r\times(\boldsymbol{D}'+\boldsymbol{P}_s)]=\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}+\frac{\partial}{\partial \mathrm{t}}[\boldsymbol{D}'+\boldsymbol{P}_s] \tag{29d}$$

where $\boldsymbol{v}(t)$ is only time dependent, but $\boldsymbol{v}_r=\boldsymbol{v}_r(\boldsymbol{r},t)$ is more general. Note that equations (29a)–(29d) are regarded as the general MEs for shape-deformable, mechano-driven, slow-moving media at an arbitrary velocity field. This full MEs-f-MDMS describes the coupling among three fields: mechano–electricity–magnetism. The law of charge conservation is

$$\nabla\cdot[\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}]+\frac{\partial}{\partial t}\rho_{\mathrm{f}}=0 \tag{29e}$$

The physical meaning of each term in equation (29e) is explained as follows (see Figure 5), where $\boldsymbol{v}$ is the moving velocity of the origin of the moving reference frame S' in the rest frame S; $\boldsymbol{v}_r$ is the relative movement velocity of the medium in the S' frame; $\boldsymbol{P}_s$ is the polarization introduced due to the relative movement of the objects in the S' frame if there is more than one object present.

We note that the MEs-f-MDMS is utilized for the space inside of a moving medium, while outside the medium in vacuum space, the governing equation is the classical MEs (Figure 5). The solutions of the two sets of equations meet at the media boundaries as governed by boundary conditions [5], [13]

$$[\boldsymbol{D}_2'-\boldsymbol{D}_1'+\boldsymbol{P}_{s2}-\boldsymbol{P}_{s1}]\cdot\boldsymbol{n}=\sigma_{\mathrm{f}} \tag{30a}$$

$$[\boldsymbol{B}_2-\boldsymbol{B}_1]\cdot\boldsymbol{n}=0 \tag{30b}$$

$$\boldsymbol{n}\times[\boldsymbol{E}_2-\mathbf{E}_1+\boldsymbol{v}_{r2}\times\boldsymbol{B}_2-\boldsymbol{v}_{r1}\times\mathbf{B}_1)]=0 \tag{30c}$$

$$\boldsymbol{n}\times[\boldsymbol{H}_2-\boldsymbol{H}_1-\boldsymbol{v}_{r2}\times(\boldsymbol{D}_2'+\boldsymbol{P}_{s2})+\boldsymbol{v}_{r1}\times(\boldsymbol{D}_1'+\boldsymbol{P}_{s1})]=\boldsymbol{K}_s+\sigma_{\mathrm{f}}\boldsymbol{v}_s \tag{30d}$$

In addition, we can neglect consideration of the speed of light in the medium exceeding that of the speed of light in vacuum c_0.

VII. Conservation of Energy

The conservation of energy in the mechano-electric-magnetic coupling system is next studied. Starting from equations (29a)–(29d), the energy conservation process in this mechano-electric-magnetic coupling system is given by [5], [13]:

$$-\frac{\partial}{\partial t}u-\nabla\cdot\boldsymbol{S}=\boldsymbol{E}\cdot\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}\cdot\boldsymbol{E}+\{\boldsymbol{H}\cdot[\nabla\times(\boldsymbol{v}_r\times\boldsymbol{B})] +\boldsymbol{E}\cdot[\nabla\times(\boldsymbol{v}_r\times(\boldsymbol{D}'+\boldsymbol{P}_s))]\} \tag{31}$$

where $\boldsymbol{S}$ is the Poynting vector, representing the energy per unit time per unit area transported by the fields

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H} \tag{32}$$

and u is the energy volume density of the electromagnetic field, which can be given by

$$\frac{\partial}{\partial t}u=\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial \mathrm{t}}+\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial \mathrm{t}} \tag{33}$$

Equation (31) indicates that the decrease in the internal electromagnetic field energy within a volume plus the rate of electromagnetic wave energy radiated out of the volume surface is the rate of energy done by the field on the external free current and the free charges, plus the media spatial motion-induced change in electromagnetic energy density. And importantly, the contribution made by media movement can be regarded as a "source" for producing electromagnetic energy.

Furthermore, if the medium movement only depends on time $\boldsymbol{v}_r(t)$, e.g., solid translation, the above equations are simplified as

$$-\frac{\mathrm{D}}{\mathrm{D}t}u-\nabla\cdot\boldsymbol{S}=\boldsymbol{E}\cdot\boldsymbol{J}_{\mathrm{f}} \tag{34a}$$

with

$$\frac{\mathrm{D}}{\mathrm{D}t}u=\boldsymbol{E}\cdot\frac{\mathrm{D}\boldsymbol{D}}{\mathrm{D}t}+\boldsymbol{H}\cdot\frac{\mathrm{D}\boldsymbol{B}}{\mathrm{D}t} \tag{34b}$$

and

$$\frac{\mathrm{D}}{\mathrm{D}t}=\frac{\partial}{\partial t}-(\boldsymbol{v}_r\cdot\nabla) \tag{34c}$$

$\boldsymbol{E}\cdot\boldsymbol{J}_{\mathrm{f}}$ is a source term that transfers energy from (to) the electromagnetic field to (from) the charged medium that interacts with the field. The mechanical energy of the charged medium increases (decreases) accordingly.

VIII. Mathematical Solutions of the Expanded Maxwell's Equations

Inside the moving object, the general solution of the equations has two components: a homogeneous solution that satisfies [6], [11]

$$\nabla\cdot\boldsymbol{D}_{\mathrm{h}}'=0 \tag{35a}$$

$$\nabla\cdot\boldsymbol{B}_{\mathrm{h}}=0 \tag{35b}$$

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$$\nabla\times(\boldsymbol{E}_{\mathrm{h}}+\boldsymbol{v}_r\times\boldsymbol{B}_{\mathrm{h}})=-\frac{\partial}{\partial t}\boldsymbol{B}_{\mathrm{h}} \tag{35c}$$

$$\nabla\times(\boldsymbol{H}_{\mathrm{h}}-\boldsymbol{v}_r\times\boldsymbol{D}'_{\mathrm{h}})=\frac{\partial}{\partial t}\boldsymbol{D}'_{\mathrm{h}} \tag{35d}$$

and a special solution that satisfies

$$\nabla\cdot\boldsymbol{D}'_{\mathrm{s}}=\rho_{\mathrm{f}}-\nabla\cdot\boldsymbol{P}_s \tag{36a}$$

$$\nabla\cdot\boldsymbol{B}_{\mathrm{s}}=0 \tag{36b}$$

$$\nabla\times(\boldsymbol{E}_{\mathrm{s}}+\boldsymbol{v}_r\times\boldsymbol{B}_{\mathrm{s}})=-\frac{\partial}{\partial t}\boldsymbol{B}_{\mathrm{s}} \tag{36c}$$

$$\nabla\times[\boldsymbol{H}_{\mathrm{s}}-\boldsymbol{v}_r\times(\boldsymbol{D}'_{\mathrm{s}}+\boldsymbol{P}_s)]=\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}-\nabla\times(\boldsymbol{v}_r\times\boldsymbol{P}_s)+\frac{\partial}{\partial t}[\boldsymbol{D}'_{\mathrm{s}}+\boldsymbol{P}_s] \tag{36d}$$

Apparently, both the homogenous solution and special solution are affected by the motion of the medium.

Outside of the object in vacuum, the homogenous solution of the MEs is determined by

$$\nabla\cdot\boldsymbol{D}'_{\mathrm{h}}=0 \tag{37a}$$

$$\nabla\cdot\boldsymbol{B}_{\mathrm{h}}=0 \tag{37b}$$

$$\nabla\times\boldsymbol{E}_{\mathrm{h}}=-\frac{\partial}{\partial t}\boldsymbol{B}_{\mathrm{h}} \tag{37c}$$

$$\nabla\times\boldsymbol{H}_{\mathrm{h}}=\frac{\partial}{\partial t}\boldsymbol{D}'_{\mathrm{h}} \tag{37d}$$

The special solution is given by

$$\nabla\cdot\boldsymbol{D}'_{\mathrm{s}}=\rho_{\mathrm{f}} \tag{38a}$$

$$\nabla\cdot\boldsymbol{B}_{\mathrm{s}}=0 \tag{38b}$$

$$\nabla\times\boldsymbol{E}_{\mathrm{s}}=-\frac{\partial}{\partial t}\boldsymbol{B}_{\mathrm{s}} \tag{38c}$$

$$\nabla\times\boldsymbol{H}_{\mathrm{s}}=\boldsymbol{J}_{\mathrm{f}}+\frac{\partial}{\partial t}\boldsymbol{D}'_{\mathrm{s}} \tag{38d}$$

The total solution is a sum of the homogeneous solution and the special solution, and it needs to meet the boundary conditions as defined by equations (30a)–(30d).

If the instantaneous shape of the medium is defined by $s(\boldsymbol{r},t)=0$ and the moving trajectory of the center of the moving reference frame is defined as $\boldsymbol{r}_0(t)$ (see Figure 5), the governing equations are (35a)–(35d) and (36a)–(36d) when $\boldsymbol{r}$ is within the volume of the surface $s(\boldsymbol{r}-\boldsymbol{r}_0(t),t)=0$; otherwise, the governing equations are (37a)–(37d) and (38a)–(38d). The solutions of the two sets of equations satisfy the boundary conditions given in (30a)–(30d) at the surface defined by $s(\boldsymbol{r}-\boldsymbol{r}_0(t),t)=0$. This is the general principle for finding the numerical solutions for the entire system.

1. Perturbation theory for a general moving velocity

Although the MEs-f-MDMS provide a complete description of the electromagnetics of the system, their solutions are most important. Analytical solutions are only possible for very simple cases. For most engineering applications, numerical calculations are therefore required. Since the theory was derived for the low-speed case $v\ll c$, we can expand the full solution in the order of $\boldsymbol{v_r}$. Considering the dominant contribution made from the stationary medium case, e.g., $\boldsymbol{v_r}=0$ (the zeroth order), we can use the perturbation approach as developed in quantum mechanics for solving the MEs-f-MDMS. In the time/frequency space, the solution of the MEs-f-MDMS can be derived order by order using perturbation theory on the order of $\boldsymbol{v_r}$. The higher-order solution is received using the iteration method. More details have been covered previously [5], [12].

2. Vector potential for an object that moves as a solid translation

We now present the solution of the MEs-f-MDMS if the motion of the object is a solid translation, which indicates that $\boldsymbol{v}_{\mathrm{t}}(t)$ is space-independent, the object is a solid object and its movement is a translation without rotation. If the medium is a simple linear medium, equations (29a)–(29d) can be further derived as [6]

$$\nabla\cdot\boldsymbol{D}'=\rho' \tag{39a}$$

$$\nabla\cdot\boldsymbol{B}=0 \tag{39b}$$

$$\nabla\times\boldsymbol{E}=-\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{B} \tag{39c}$$

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}'+\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{D}' \tag{39d}$$

where

$$\rho'=\rho_{\mathrm{f}}-\nabla\cdot\boldsymbol{P}_s \tag{39e}$$

$$\boldsymbol{J}'=\boldsymbol{J}_{\mathrm{f}}+\rho_{\mathrm{f}}\boldsymbol{v}_{\mathrm{t}}+\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{P}_s \tag{39f}$$

We now define the vector potential $\boldsymbol{A}$ as follows:

$$\boldsymbol{B}=\nabla\times\boldsymbol{A} \tag{40a}$$

and a new scalar electric potential φ for electrostatics, we define

$$\boldsymbol{E}=-\nabla\varphi-\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{A} \tag{40b}$$

Substituting equations (40a) and (40b) into (39a)–(39d) and make use of the constitutive relations, we have

$$\nabla^2\boldsymbol{A}-\varepsilon\mu\frac{\mathrm{D}^2}{\mathrm{D}t^2}\boldsymbol{A}=-\mu\boldsymbol{J}' \tag{41a}$$

$$\nabla^2\varphi-\varepsilon\mu\frac{\mathrm{D}^2}{\mathrm{D}t^2}\varphi=-\frac{\rho'}{\varepsilon} \tag{41b}$$

where $\frac{\mathrm{D}^2}{\mathrm{D}t^2}=\left[\frac{\partial}{\partial t}-(\boldsymbol{v}_r\cdot\nabla)\right]\left[\frac{\partial}{\partial t}-(\boldsymbol{v}_r\cdot\nabla)\right]=\frac{\partial^2}{\partial t^2}-2(\boldsymbol{v}_r\cdot\nabla)\frac{\partial}{\partial t}+(\boldsymbol{v}_r\cdot\nabla)(\boldsymbol{v}_r\cdot\nabla)$, and the Lorentz gauge must be satisfied

$$\nabla\cdot\boldsymbol{A}+\varepsilon\mu\frac{\mathrm{D}}{\mathrm{D}t}\varphi=0 \tag{41c}$$

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These are nonhomogeneous wave equations for vector potentials $\boldsymbol{A}$ and φ, which are nonlinear differential equations. Equations (41a) and (41b) have a similar format as the Navier-Stokes equations for fluid except with the 3 space variables plus time. They are simple approximations for isotropic media. The total solutions may have to be solved numerically, and the total solutions must satisfy the boundary conditions as defined in equations (30a)–(30d).

We express the MEs-f-MDMS equations in tensor format. We use the classical expressions of the following quantities for electrodynamics, the anti-symmetric strength tensor of the electromagnetic field [5]

$$F^{\alpha\beta} = \xi^{\alpha} A^{\beta} - \xi^{\beta} A^{\alpha} \tag{42a}$$

$$F_{\alpha\beta} = \xi_{\alpha} A_{\beta} - \xi_{\beta} A_{\alpha} \tag{42b}$$

where α, $\beta = (1,2,3,4)$, and the newly defined operators are

$$\xi^{\alpha} = \left(\frac{1}{c}\frac{\mathrm{D}}{\mathrm{D}t}, -\nabla\right) \tag{43a}$$

$$\xi_{\alpha} = \left(\frac{1}{c}\frac{\mathrm{D}}{\mathrm{D}t^2}, \nabla\right) \tag{43b}$$

$$A^{\alpha} = (c\varphi, \boldsymbol{A}) \tag{43c}$$

$$A_{\alpha} = (c\varphi, -\boldsymbol{A}) \tag{43d}$$

We prove

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} \tag{44a}$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} \tag{44b}$$

where $c = c_{\mathrm{m}} = 1/(\mu\varepsilon)^{1/2}$. Equations (39a)–(39e) can be restated as

$$\xi_{\alpha} F^{\alpha\beta} = \mu J^{\beta} \tag{45}$$

where $J^{\beta} = (c\rho', \boldsymbol{J}')$. This is Maxwell's equation for a mechano-driven system. Note that equation (45) is the same as that for the classical MEs except the operator ∂_{α} is replaced by ξ_{α}. The density of the Lagrangian for the electromagnetic field is given by

$$\Lambda = F^{\alpha\beta} F_{\alpha\beta} + \mu J^{\alpha} A_{\alpha} \tag{46}$$

IX. Nanoscale Electromagnetic Phenomena and Boundary Conditions

Here, we review additional progress made in the field for expanding the boundary condition for nanoscale media, such as nanoparticles, which may not be directly related to what we have presented above. Macroscopic electromagnetic boundary conditions (EMBCs) have been proposed for over a century and have a wide range of applications in physics. The EMBCs are built based on the abrupt interface assumption by neglecting the intrinsic electronic length scales associated with interfaces; more specifically, the integral contribution of the traditional electromagnetic field along the sidewall of the integrating box is neglected [39]. This treatment results in considerable discrepancies when it is utilized to describe electromagnetic phenomena in systems with nanoscale feature sizes, such as surface photoexcitation and nanoplasmonics. To reconcile classical predications and experimental observations, the Feibelman d parameters, which are interfacial response functions (IRFs), were developed by Feibelman to capture nanoscale electromagnetic phenomena [39]–[41]. Since then, two forms of nanoscale electromagnetic boundary conditions have been proposed according to these d parameters.

Assuming an interface that is formed by two isotropic bulk materials with different permittivity and permeability, there simultaneously exists a transition layer in which the permittivity and permeability change continuously from one material to the other. Through MEs and the contributions of the transition layer as the first-order perturbation of the classical EMBCs, the nanoscale EMBCs are deduced [40] as follows:

$$[\![\boldsymbol{E}_{\parallel}]\!] = -d_{\perp}\nabla_{\parallel}[\![E_{\perp}]\!] - i\omega b_{\parallel}[\![\boldsymbol{B}_{\parallel}]\!]\times\hat{n} \tag{47a}$$

$$[\![\boldsymbol{H}_{\parallel}]\!] = -b_{\perp}\nabla_{\parallel}[\![H_{\perp}]\!] + i\omega d_{\parallel}[\![\boldsymbol{D}_{\parallel}]\!]\times\hat{n} \tag{47b}$$

$$[\![D_{\perp}]\!] = d_{\parallel}\nabla_{\parallel}\cdot[\![\boldsymbol{D}_{\parallel}]\!] \tag{47c}$$

$$[\![B_{\perp}]\!] = b_{\parallel}\nabla_{\parallel}\cdot[\![\boldsymbol{B}_{\parallel}]\!] \tag{47d}$$

where $[\![W_i]\!] = W_i(z_1) - W_i(z_2)$ represents the discontinuity of the tangential ($i = \parallel$) / normal ($i = \perp$) component of the field $\overrightarrow{W}$ across the interface. $\nabla_{\parallel} = \frac{\partial}{\partial x}\hat{n}_x + \frac{\partial}{\partial y}\hat{n}_y$. $\widehat{n}$ is the unit vector perpendicular to the interface from one medium to the other. From the above equations, it is observed that the discontinuity of the electromagnetic field tangential component $\boldsymbol{E}_{\parallel}(\boldsymbol{H}_{\parallel})$ across the interface is coupled not only with the normal component $E_{\perp}(H_{\perp})$ but also with the inductive field component $\boldsymbol{D}_{\parallel}(\boldsymbol{B}_{\parallel})$. The discontinuity of the inductive field $D_{\perp}(B_{\perp})$ is proportional to the in-plane divergence of the corresponding tangential component $\boldsymbol{D}_{\parallel}(\boldsymbol{B}_{\parallel})$. We now focus on the physical meanings of interfacial response functions. $d_{\perp}$ represents the centroid of the interface-induced polarization charge, and $d_{\parallel}$ is the centroid of the normal derivative of the tangential current, which are expressed as

$$d_{\perp} = \frac{\int_{-\infty}^{\infty} z\rho_{\mathrm{ind}}\mathrm{d}z}{\int_{-\infty}^{\infty}\rho_{\mathrm{ind}}\mathrm{d}z} \tag{48a}$$

$$d_{\parallel} = \frac{\int_{-\infty}^{\infty} z\frac{\mathrm{d}j_{\mathrm{py}}}{\mathrm{d}z}\mathrm{d}z}{\int_{-\infty}^{\infty}\frac{\mathrm{d}j_{\mathrm{py}}}{\mathrm{d}z}\mathrm{d}z} \tag{48b}$$

Note that ρ_{ind} is the surface-induced polarization charge density, and j_{py} represents the tangential polarization current on the surface. $b_\perp$ represents the centroid of the equivalent magnetization charge density, and $b_\parallel$ represents the centroid of the equivalent magnetization current density; they are rewritten as

$$b_\perp = \frac{\int_{-\infty}^{\infty} z\rho_m dz}{\int_{-\infty}^{\infty} \rho_m dz} \tag{49a}$$

and

$$b_\parallel = \frac{\int_{-\infty}^{\infty} z j_m dz}{\int_{-\infty}^{\infty} j_m dz} \tag{49b}$$

respectively.

Moreover, based on the definition of electric and magnetic dipole moments, the interface-induced dipole moments are introduced; thus, the nanoscale EMBCs are rewritten as [40]

$$[\![\boldsymbol{E}_\parallel]\!] = -\left(\frac{-1}{\varepsilon_0}\nabla\times\boldsymbol{\pi}_\perp + \mu_0\frac{\partial \boldsymbol{m}_\parallel}{\partial t}\right)\times\hat{n} \tag{50a}$$

$$[\![\boldsymbol{H}_\parallel]\!] = \left(\nabla\times\boldsymbol{m}_\perp + \frac{\partial \boldsymbol{\pi}_\parallel}{\partial t}\right)\times\hat{n} \tag{50b}$$

$$[\![D_\perp]\!] = -\nabla_\parallel\cdot\boldsymbol{\pi}_\parallel \tag{50c}$$

$$[\![B_\perp]\!] = -\mu_0\nabla_\parallel\cdot\boldsymbol{m}_\parallel \tag{50d}$$

where $\boldsymbol{\pi}_\perp = \varepsilon_0 d_\perp [\![E_\perp]\!]\hat{n}$ and $\boldsymbol{\pi}_\parallel = -d_\parallel [\![\boldsymbol{D}_\parallel]\!]$ for the electric dipole and $\boldsymbol{m}_\perp = b_\perp [\![H_\perp]\!]\hat{n}$ and $\boldsymbol{m}_\parallel = -b_\parallel [\![\boldsymbol{B}_\parallel]\!]$ for the magnetic dipole. Therefore, the interface with a transition layer can be regarded as the abrupt interface with interface-induced electric and magnetic dipole moments. A general conclusion is that the nanoscale EMBCs are different based on the results of assuming polarization and magnetization across the abrupt interface in the basic physical model; they are obtained from the transition interface model with inhomogeneous electromagnetic field-induced polarization and magnetization. As stated, we have derived the macroscopic boundary conditions of the MEs-f-MDMS. Undoubtedly, these EMBCs can be extended to obtain nanoscale boundary conditions through a special interface mode in which the intrinsic electronic length scales must be considered. Another important aspect is that the above macroscopic and nanoscale EMBCs are proposed for classical MEs, which are initially built for media having a fixed boundary and volume, particularly in a stationary state. However, these assumptions are typically not focused on in the general literature.

X. Summary and Outlook

This paper systematically reviews the recent progress in developing Maxwell's equations for a mechano-driven media system (MEs-f-MDMS). These equations are utilized to describe the electromagnetism of multimoving media. The basic theoretical framework, formulation and solutions of the equations are fully elaborated in reference to the classical MEs. The main insights of this review are summarized as follows [5], [13]:

a) Based on the integral forms of the four physics laws and in the Galilean space-time, the MEs-f-MDMS are derived to describe the electrodynamics of slow-moving media that may move with acceleration.

b) ME-f-MDMS is typically used to reveal the dynamics of an electromagnetic field for a general case, in which the medium has a time-dependent volume, shape, and boundary and may move in an arbitrary velocity field $\boldsymbol{v}_r(\boldsymbol{r},t)$ in a noninertial system.

c) By neglecting the relativity effect, the expanded MEs-f-MDMS are applicable to reveal the electrodynamics of a mechanical force-electricity-magnetism system.

d) The total energy of electricity and magnetism is not conserved since an external mechanical energy is input; however, the total energy of the closed mechano-driven media system is conserved.

e) The charged moving media are regarded as the sources for generating electromagnetic radiation (a motion-generated electromagnetic field). The created electromagnetic wave within the moving media can be described by the expanded MEs-f-MDMS, and its propagation in space satisfies the standard MEs and special relativity; they meet at the medium interface as governed by the boundary conditions.

f) Distinct from the methods of relativity electrodynamics in which the electromagnetic fields in the observation frame and the comoving frame are correlated by the Lorentz transformation, the expanded MEs-f-MDMS are for the case in which the observation is in the observation frame, while the media are moving at complex velocities along various trajectories. In other words, all fields are expressed in the variables in the observation frame, which is more useful for describing engineering problems.

g) Because the speed of light inside media c_m is generally lower than c_0, there is no need to consider the case of exceeding the speed of light in vacuum c_0 even when the medium is moving. Once the electromagnetic wave is generated from the mechano-driven media system, its trajectory outside the medium is governed by the classical MEs, regardless of whether the media are moving or not.

h) The expanded MEs-f-MDMS can describe the electrodynamics of fluid/liquid media because it has been proven that these equations can describe the electromagnetism of the mechano-driven system in the noninertial frame with acceleration and even time-dependent volume, shape, and boundary.

i) If the medium moves at a constant velocity so that $\boldsymbol{v}$ = constant and $\boldsymbol{v}_r = 0$, equations (29a)–(29d) resume the format of the classical MEs, so there is no logical inconsistency with the existing theory.

In comparison to the classical Maxwell equations, the

MEs-f-MDMS has made the following expansions:

1) Accelerated motion in a noninertia reference frame vs. uniform motion along a straight line in the inertia reference frame;

2) Electromagnetism that includes the Feynman "anti-flux rule" examples vs. that excludes such cases;

3) Electrodynamics for multimoving media vs. that for one moving medium; and

4) The entire field (both near field and far-field) electrodynamics vs. the far-field plus partial near-field electrodynamics.

Much existing research focuses on the far-distance transmission and reflection of electromagnetic waves, such as wireless communication and propagation, antennas, and radar, demonstrating the special solutions of MEs. The effects from the motion status of the electric current source and the mechanical action for generating the current on the distribution of electromagnetic fields in the vicinity have been ignored. Such a near-field effect can be important for new technological applications in short-range wireless sensing. MEs-f-MDMS provide an accurate and practical method to systematically investigate both far-field electromagnetic behavior and near-field electromagnetic behavior for engineering applications.

Importantly, we note that *the covariance of the MEs holds exactly for the electromagnetic phenomena in vacuum!* If there are moving objects/media in space at any speed, the Lorentz transformation may not be applicable for treating the electromagnetic behavior because the covariance of the MEs may not hold in this case; therefore, MEs-f-MDMS is probably the most effective approach if the moving velocity is low. We have also discussed the revision of the Lorentz transformation from the vacuum case to the medium case, which is subjected to further studies.

MEs-f-MDMS is a unification of the theory for electromagnetic generators/motors and the theory of electromagnetic waves (Figure 6). The theory of electromagnetic generators relies on the rotation of a rotor to cut through a magnetic field so that the mechanical energy is converted into electric power. Most important is that the electric current and voltage are carried by the conduction coil, which disregards electromagnetic waves radiated to the space nearby. MEs consider these electromagnetic waves that radiate when an oscillating current is supplied. Once the observation point is close to the electromagnetic generator, near to which the rotation of the rotor is quite dominant, the MEs can predict the electromagnetic behavior arising from the current conducted in the metal wire, but they may not precisely predict the effect of the rotating rotor on the field distributed nearby. This is why we need the MEs-f-MDMS.

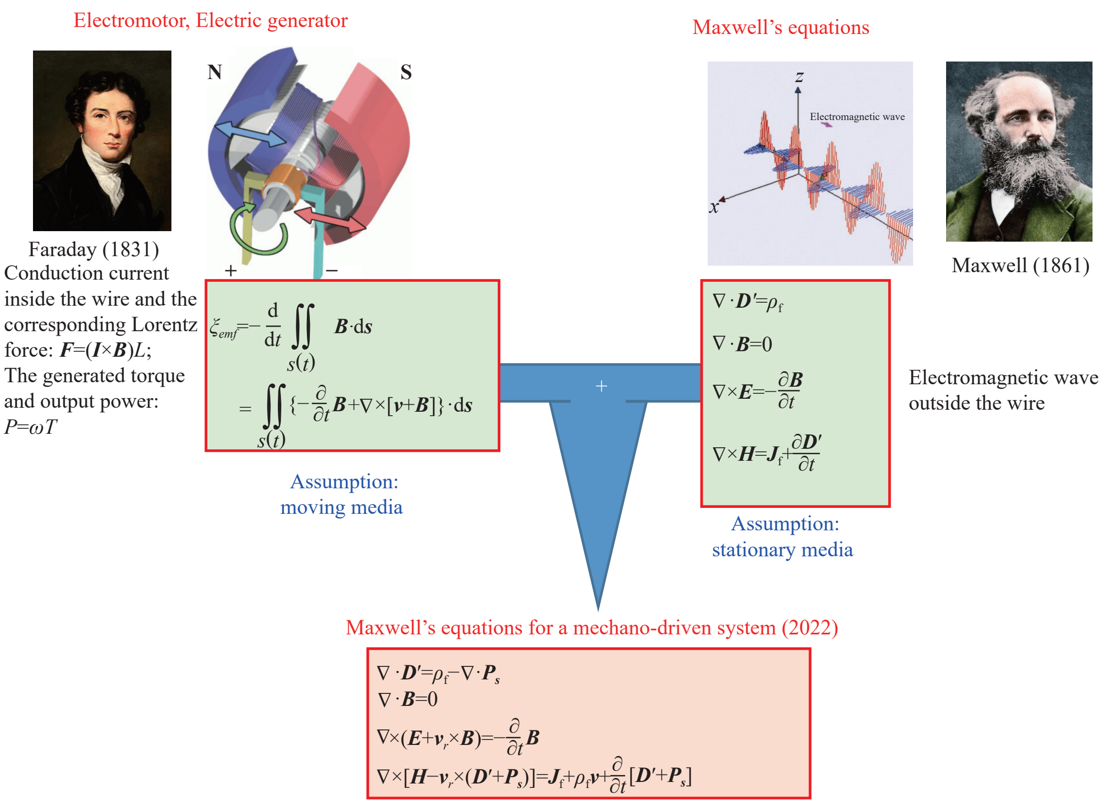

Figure 6 The MEs-f-MDMS are a unification of the theory for electromagnetic generator/motor and the theory of electromagnetic waves, so that the field in the entire space can be calculated. MEs-f-MDMS are likely to make a key difference in the regions near the moving objects, which may not be fully covered by the classical MEs. This is the contribution of the MEs-f-MDMS to the fundamentals of electrodynamics [13].

Because MEs have tremendously impacted modern science and technology, many scientists are still now focusing on their new development [42], [43]. Mechano-driven polarization $\boldsymbol{P}_s$, which is also called the Wang term, was first introduced in MEs to quantify the output performance of TENGs. ME-f-MDMS describes the electromagnetism of media systems in the noninertial frame with a time-dependent volume, shape, and boundary by neglecting the relativistic effect. MEs-f-MDMS is not an alternative to MEs but is precisely for developing engineering electromagnetism toward today's technology needs. Emerging potential fields directly or indirectly impacted by MEs-f-MDMS could include wireless communication and propagation, antennas, radar, radar cross-section (RCS) analysis and design, electromagnetic compatibility (EMC) and electromagnetic interference (EMI) analysis and design (Figure 7), which could inspire new discoveries with unprecedented technological advances.

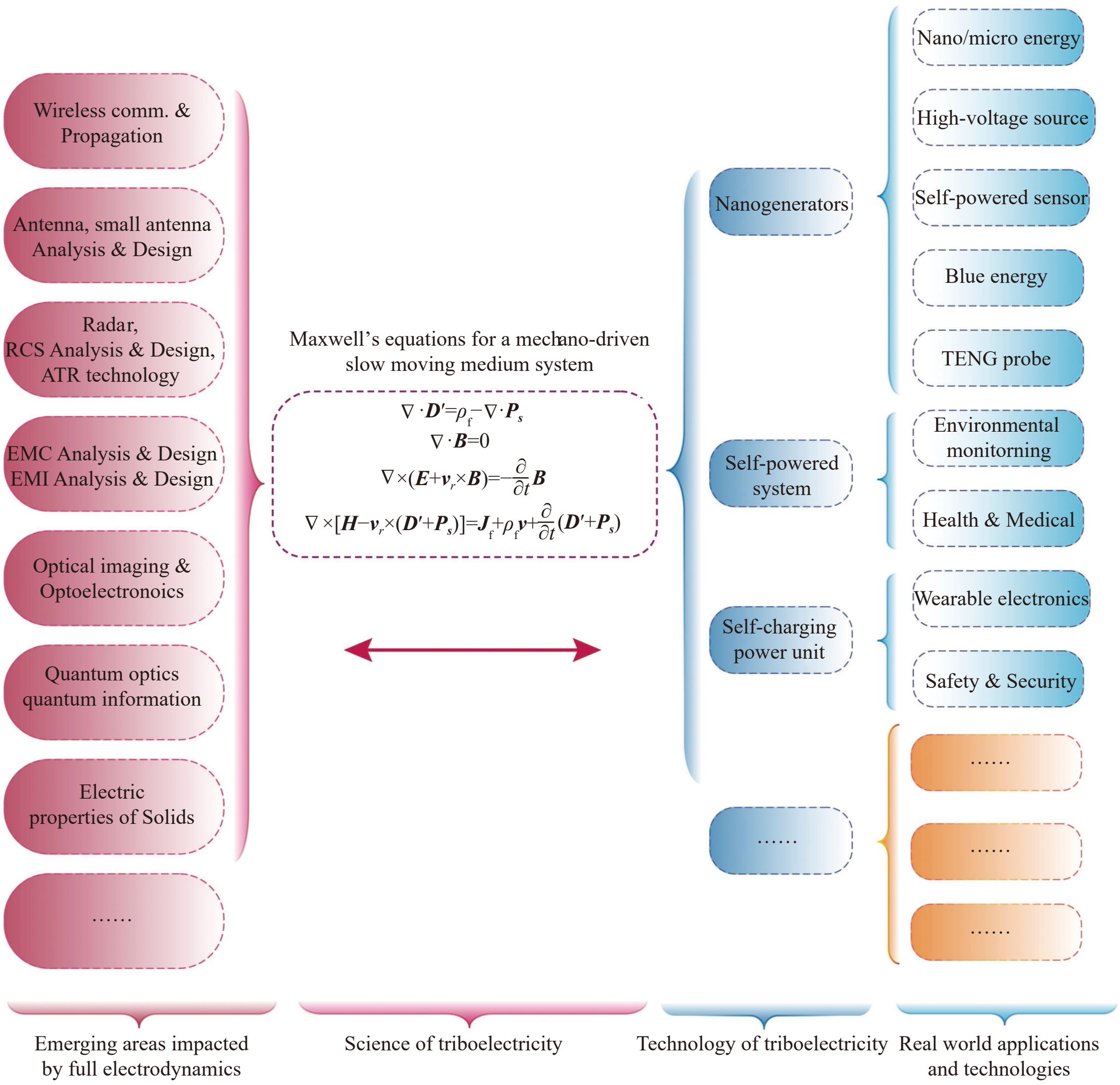

Figure 7 Emerging areas that may be impacted by f MEs-f-MDMS, with potential technological importance [13].

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing Financial Interests

The authors declare no competing financial interests.

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